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The Math Forum

Ask Dr. Math: FAQ

The Calendar and the Days of the Week

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What years are leap years? What day of the week will it be, a year from today? How do I find the day of the week for any date? How do I find a calendar for any year? How common are Friday the 13ths?

What years are leap years?

It takes the earth about 365.2422 days to go around the sun, but a normal calendar year is only 365 days. The extra fraction of a day adds up: circling the sun four times takes 1460.9688 days, but four calendar years would only be 1460 days. That .9688 is almost a whole day, so every four years we add an extra day to our calendar, February 29. We call that year leap year. To make things easier, leap years are always divisible by four: 2004 and 2008 will both be leap years.

For hundreds of years, people used a calendar called the Julian calendar that followed this rule, adding a leap year every four years. However, because .9688 isn't exactly a whole day, the Julian calendar slowly began to disagree with the real seasons. In 1582, Pope Gregory fixed this problem by ordering everyone to use a new set of rules. These rules are named the Gregorian calendar, after him. They work like this:

The Gregorian Calendar

Rule Examples

Every fourth year is a leap year. 2004, 2008, and 2012 are leap years.

However, every hundredth year is not a leap year. 1900 and 2100 are not leap years.

Every four hundred years, there's a leap year after all. 2000 and 2400 are leap years.

People in English-speaking countries didn't start using the Gregorian calendar until 1752. Some countries, such as Iran, still use other systems.

From the Dr. Math archives:

Why Do We Have Leap Year?

Converting Gregorian Date to Gregorian Days

From the Web:

Leap Year, Calendar, Julian Calendar, and Gregorian Calendar,

from Eric Weisstein's World of Astronomy

Calendopaedia - The Encyclopaedia of Calendars, Michael Astbury

Dear Marilyn, About Leap Years, Frank Morgan's Math Chat

Calendars, L.E. Doggett

Iranian Calendars, Kelley L. Ross

What day of the week will it be, a year from today?

Suppose that today is February 13, 2053. The day of the week is a Thursday. One week (seven days) from today, on February 20, 2053, it will be Thursday again. After two weeks (fourteen days), it will be Thursday once more. We need to figure out how many weeks and days will have passed after a year. 2053 is not a leap year, so 365 days will pass between February 13, 2053 and February 13, 2054. Let's divide by 7 to find out how many weeks that is: 365 / 7 = 52, remainder 1, or fifty-two weeks with one day left over. Fifty-two weeks after February 13, 2053 is yet another Thursday, so fifty-two weeks and one day later must be a Friday.

We can use the same method for any date, but we have to be careful. Because some years are leap years, a year from today might be 366 days and not 365. For instance, there are 366 days between Saturday, November 20, 2055 and November 20, 2056, because 2056 is a leap year and February 29, 2056 lies between the two Novembers.

From the Dr. Math archives:

Counting Days of the Week

Birthdays Every Year

How do I find the day of the week for any date?

There are two popular formulas that you can use to find the day of the week for a given date. You should be careful when you use these formulas, though, because they only work for the Gregorian calendar. (People in English-speaking countries used a different calendar before September 14, 1752.)

Zeller's Rule

The following formula is named Zeller's Rule after a Reverend Zeller. [x] means the greatest integer that is smaller than or equal to x. You can find this number by just dropping everything after the decimal point. For example, [3.79] is 3. Here's the formula:

f = k + [(13\*m-1)/5] + D + [D/4] + [C/4] - 2\*C.

k is the day of the month. Let's use January 29, 2064 as an example. For this date, k = 29.

m is the month number. Months have to be counted specially for Zeller's Rule: March is 1, April is 2, and so on to February, which is 12. (This makes the formula simpler, because on leap years February 29 is counted as the last day of the year.) Because of this rule, January and February are always counted as the 11th and 12th months of the previous year. In our example, m = 11.

D is the last two digits of the year. Because in our example we are using January (see previous bullet) D = 63 even though we are using a date from 2064.

C stands for century: it's the first two digits of the year. In our case, C = 20.

Now let's substitute our example numbers into the formula.

f = k + [(13\*m-1)/5] + D + [D/4] + [C/4] - 2\*C

= 29 + [(13\*11-1)/5] + 63 + [63/4] + [20/4] - 2\*20

= 29 + [28.4] + 63 + [15.75] + [5] - 40

= 29 + 28 + 63 + 15 + 5 - 40

= 100.

Once we have found f, we divide it by 7 and take the remainder. Note that if the result for f is negative, care must be taken in calculating the proper remainder. Suppose f = -17. When we divide by 7, we have to follow the same rules as for the greatest integer function; namely we find the greatest multiple of 7 less than -17, so the remainder will be positive (or zero). -21 is the greatest multiple of 7 less than -17, so the remainder is 4 since -21 + 4 = -17. Alternatively, we can say that -7 goes into -17 twice, making -14 and leaving a remainder of -3, then add 7 since the remainder is negative, so -3 + 7 is again a remainder of 4.

A remainder of 0 corresponds to Sunday, 1 means Monday, etc. For our example, 100 / 7 = 14, remainder 2, so January 29, 2064 will be a Tuesday.

The Key Value Method

This method uses codes for different months and years to speed up the calculation of the day of the week. You might even be able to memorize the codes. We'll use December 16, 2482 as an example.

Take the last 2 digits of the year. In our example, this is 82.

Divide by 4, and drop any remainder. 82 / 4 = 20, remainder 2, so we think "20."

Add the day of the month. In our example, 20 + 16 = 36.

Add the month's key value, from the following table.

Jan Feb Mar Apr May June July Aug Sept Oct Nov Dec

1 4 4 0 2 5 0 3 6 1 4 6

The month for our example is December, with a key value of 6. 36 + 6 = 42.

If your date is in January or February of a leap year, subtract 1. We're using December, so we don't have to worry about this step.

Add the century code from the following table. (These codes are for the Gregorian calendar. The rule's slightly simpler for Julian dates.)

1700s 1800s 1900s 2000s

4 2 0 6

Our example year is 2482, and the 2400s aren't in the table. Luckily, the Gregorian calendar repeats every four hundred years. All we have to do is add or subtract 400 until we have a date that is in the table. 2482 - 400 = 2082, so we look at the table for the 2000s, and get the code 6. Now we add this to our running total: 42 + 6 = 48.

Add the last two digits of the year. 48 + 82 = 130.

Divide by 7 and take the remainder. This time, 1 means Sunday, 2 means Monday, and so on. A remainder of 0 means Saturday. 130 / 7 = 18, remainder 4, so December 16, 2482 will be on the fourth day of the week-- Wednesday.

From the Dr. Math archives:

Formula for the Day of the Week

Deriving Zeller's Rule

Formula for the Day of the Month

Formula for the First Day of a Year

This formula is slightly different from Zeller's formula. Can you see why?

Calendar Trick

Day Finding

Pascal Calendar Program

Years That Start on Sunday

From the Web:

Calendrical Calculations, Nachum Dershowitz and Edward M. Reingold

A book about converting between different calendars. A Java applet lists the date in each calendar for a given day, and displays a Gregorian calendar for the entire month.

Weekday, Eric Weisstein's World of Astronomy

sci.math FAQ: Day of Week

Identical Calendars for 2000, Frank Morgan's Math Chat

Day of the Week for Any Date, Jim Loy

How do I find a calendar for any year?

In any year, January 1 could be on any of the seven weekdays, and the year might or might not be a leap year. This means that there are only 7 \* 2 = 14 possible ways to make a calendar that has every date listed under the appropriate day of the week. You can make a bigger calendar that tells you which yearly calendar to use for any given year. This kind of calendar is called a perpetual calendar. The pattern of weekdays in the Gregorian calendar repeats every four hundred years, because there are exactly 20 871 weeks in 400 Gregorian years. (100 \* (365\*3 + 366) - 3 = 146 097 days, and 146 097 / 7 = 20 871 weeks.) Because the pattern repeats, perpetual calendars only have to cover four hundred years, not all of time.

From the Dr. Math archives:

Different Yearly Calendars

How many different possible calenders are there? How many years does it take before each calendar is used at least once?

Perpetual Calendar

How do you figure the perpetual calendar?

From the Web:

Summary of how to use Herschel's Perpetual Calendar, Ron Knott

Perpetual Calendars, Telmo Ghiorzi

Print and make your own perpetual calendar.

How common are Friday the 13ths?

You can use a perpetual calendar to work out how many Friday the 13ths there are for every possible pattern of weekdays in a year. It turns out that every year has at least one Friday the 13th. The largest possible number of Friday the 13ths in one year is three. Surprisingly, it turns out that a 13th is slightly more likely to fall on a Friday than any other day. This happens because the leap year rules make some patterns of weekdays happen more often than others. The winning patterns include many Friday the 13ths. Here's a table from Eric Weisstein's World of Astronomy that shows how often the 13th falls on each day of the week, over four hundred years.

Day Number of 13ths Percent

Sunday 687 14.31%

Monday 685 14.27%

Tuesday 685 14.27%

Wednesday 687 14.31%

Thursday 684 14.25%

Friday 688 14.33%

Saturday 684 14.25%

From the Dr. Math archives:

How Many Friday the 13ths

Friday the 13ths in a Year

Years That Start on Sunday

30th of the Month

Good Friday on the 13th

From the Web:

Friday the Thirteenth, Eric Weisstein's World of Astronomy

Friday the 13th, Jim Wilson

A pair of suggested problems about Friday the 13th.

- Ursula Whitcher, for the Math Forum

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